

(2,2) SUPERGRAVITY IN THE LIGHT-CONE GAUGE

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ABSTRACT

Starting with the prepotential description of two-dimensional $(2, 2)$ supergravity we use local supersymmetry transformations to go to light-cone gauge. We discuss properties of the theory in this gauge and derive Ward identities for correlation functions defined with respect to the induced supergravity action.

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1 Introduction

In 1987 Polyakov [1] examined two-dimensional induced gravity and pointed out the existence of special features when the theory is studied in light-cone gauge. In particular he derived a set of Ward identities associated with the anomalous symmetries that the theory possesses, and showed that these Ward identities correspond to a “hidden” $SL(2, C)$ symmetry. Subsequently these results were extended to $(1, 0)$ and $(1, 1)$ two-dimensional induced supergravity [2] and $(2, 0)$ induced supergravity [3]. More recently, the Ward identities were used to further study properties of correlation functions [5] and to determine the dressing of β -functions by induced gravity [4]. In the supergravity case, similar studies were carried out by straightforward perturbation theory [6].

For two-dimensional $(2, 2)$ supergravity the exact results leading to the Ward identities have not been available, primarily because a formulation of the theory in terms of unconstrained superfields had been lacking, not only in a general gauge, but even in light-cone gauge. Recently, we have solved the constraints of $(2, 2)$ supergravity and obtained a complete description in terms of unconstrained prepotentials [7]. With this solution at hand, it is possible to discuss the situation in light-cone gauge, and derive appropriate Ward identities. This is the purpose of our paper.

Unlike previously studied cases, where reaching light-cone gauge is a fairly straightforward matter (but not quite as straightforward in the $(0, 2)$ case as the authors of ref. [3] thought ¹), $(2, 2)$ supergravity presents some problems, primarily because the solution to the constraints is highly nonlinear. Furthermore, as we shall see, whereas in other cases the unconstrained light-cone prepotential is a general $(1, 0)$, $(1, 1)$ or $(0, 2)$ superfield, here the light-cone prepotential is not a full $(2, 2)$ superfield; some of its lower components are eliminated by a Wess-Zumino gauge choice. As a consequence, the nonlinearity of the theory disappears in light-cone gauge, and the Ward identities are as simple as in previously studied cases.

Our paper is organized as follows: in Section 2 we review the solution of the constraints obtained in ref. [7]. In Section 3 we discuss the gauge transformations of the prepotentials. In Section 4 we use these transformations to reach light-cone gauge, and in Section 5 we examine the solution of the constraints in this gauge. Section 6 studies the light-cone gauge “residual” transformations under which the induced action changes by a local (anomalous) term. In Section 7 we use these transformations to derive the Ward identities. We also present some simple checks of these identities, but the study of the “hidden” symmetry that these identities represent, as well as further applications, will be postponed to a future publication.

¹ A careful examination of the gauge transformations considered by the authors of these references shows that unlike the $(1, 0)$ or $(1, 1)$ cases, some components of the vielbein cannot be straightforwardly gauged away; rather they are set to zero by the light-cone gauge constraints.

2 Solution of the (2,2) constraints

We summarize in this section the pertinent results of ref. [7]. We are considering here the $U_A(1)$ version of (2, 2) supergravity obtained by setting $\Sigma_\alpha = 0$. The spinorial covariant derivatives are defined by

$$\begin{aligned}\nabla_\alpha &= E_\alpha + \Phi_\alpha \Lambda + \tilde{\Phi}_\alpha \tilde{\Lambda} \\ &= E_\alpha + \Omega_\alpha M + \Gamma_\alpha \overline{M}\end{aligned}\tag{2.1}$$

with $\alpha = \pm$, and corresponding expressions for the complex conjugate spinorial derivatives as well as the vectorial derivatives. Here $\Lambda = M + \overline{M}$ and $\tilde{\Lambda} = -i(M - \overline{M})$ are the Lorentz and $U(1)$ generators, respectively.

The vielbein is given by

$$E_A = E_A{}^M \partial_M .\tag{2.2}$$

Torsions and curvatures are defined as usual by

$$[\nabla_A, \nabla_B] = T_{AB}{}^C \nabla_C + R_{AB} M + \overline{R}_{AB} \overline{M} .\tag{2.3}$$

They satisfy constraints which can be described by the following anticommutators

$$\begin{aligned}\{\nabla_+, \nabla_+\} &= 0 & , & \quad \{\nabla_-, \nabla_-\} = 0 \\ \{\nabla_+, \nabla_+\} &= i\nabla_{\neq} & , & \quad \{\nabla_-, \nabla_-\} = i\nabla_{=} \\ \{\nabla_+, \nabla_-\} &= -\overline{R}\overline{M} \\ \{\nabla_+, \nabla_-\} &= 0\end{aligned}\tag{2.4}$$

as well as their complex conjugates.

The solution of the constraints is obtained in terms of the “hat” differential operators

$$\hat{E}_\pm = e^{-H} D_\pm e^H \quad , \quad H = H^m i\partial_m\tag{2.5}$$

with

$$E_+ \equiv e^{\tilde{S}}(\hat{E}_+ + A_+{}^- \hat{E}_-) \quad , \quad E_- \equiv e^{\tilde{S}}(\hat{E}_- + A_-{}^+ \hat{E}_+)\tag{2.6}$$

and

$$e^{\tilde{S}} = e^{\bar{\sigma}} \frac{\left[1 \cdot e^{-\overleftarrow{H}}\right]^{-\frac{1}{2}}}{[1 - A_+{}^- A_-{}^+]^{\frac{1}{2}}} E^{-\frac{1}{2}}\tag{2.7}$$

where $\bar{\sigma}$ is a covariantly antichiral superfield and \overleftarrow{H} indicates that the differential operator in $H^m i\partial_m$ acts on objects to its left. The A ’s, as well as the connections $\Omega_\alpha, \Gamma_\alpha$, are given explicitly in ref. [7] in terms of H^m and S, \tilde{S} . The unconstrained real vector superfield H^m and the chiral scalar superfield σ are the prepotentials of (2, 2) supergravity.

3 Transformation laws for the (2,2) prepotentials

In obtaining the solution of ref. [7] we implicitly made use of the usual K invariance of the constraints on the covariant derivatives as well as some of the Λ invariance which appears as a consequence of their solution. These invariances allow us to eliminate the spinorial superfields H^α , $H^{\dot{\alpha}}$, as well as the imaginary part of H^m . We are left with further Λ invariance which we discuss in the present section. The discussion follows very much the corresponding discussion for the 4-dimensional $N = 1$ case [8]. However some details are different because we avoid the explicit introduction of “chiral representation”.

Covariantly chiral and antichiral scalar superfields Φ , $\bar{\Phi}$ are defined by the conditions $\nabla_{\pm}\Phi = \nabla_{\pm}\bar{\Phi} = 0$ or equivalently $\hat{E}_{\dot{\alpha}}\Phi = 0$, $\hat{E}_{\alpha}\bar{\Phi} = 0$. They may be expressed in terms of ordinary chiral and antichiral superfields by

$$\Phi = e^H \phi = e^H \phi e^{-H} \quad , \quad \bar{\Phi} = e^{-H} \bar{\phi} = e^{-H} \bar{\phi} e^H \quad . \quad (3.1)$$

The kinetic action for these superfields is

$$\begin{aligned} S &= \int d^2x d^4\theta E^{-1} \bar{\Phi} \Phi = \int d^2x d^4\theta E^{-1} (e^{-H} \bar{\phi}) (e^H \phi) \\ &= \int d^2x d^4\theta E^{-1} (e^{-H} \bar{\phi}) e^{-H} (e^{2H} \phi) = \int d^2x d^4\theta E^{-1} e^{-H} (\bar{\phi} e^{2H} \phi) \\ &= \int d^2x d^4\theta E^{-1} e^{\tilde{H}} (\bar{\phi} e^{2H} \phi) \end{aligned} \quad (3.2)$$

where, in the last line, we have performed an integration by parts. An equivalent expression is

$$S = \int d^2x d^4\theta E^{-1} e^{-\tilde{H}} (\phi e^{-2H} \bar{\phi}) \quad . \quad (3.3)$$

Ordinary chiral and antichiral superfields transform under superspace (coordinate) transformations as

$$\phi \rightarrow e^{i\Lambda} \phi \quad , \quad \bar{\phi} \rightarrow e^{i\bar{\Lambda}} \bar{\phi} \quad (3.4)$$

with

$$\begin{aligned} \Lambda &= \Lambda^m i \partial_m + \Lambda^\alpha i D_\alpha + \Lambda^{\dot{\alpha}} i D_{\dot{\alpha}} \\ \bar{\Lambda} &= \bar{\Lambda}^m i \partial_m + \bar{\Lambda}^\alpha i D_\alpha + \bar{\Lambda}^{\dot{\alpha}} i D_{\dot{\alpha}} \quad . \end{aligned} \quad (3.5)$$

The action in (3.2) will be invariant provided that, correspondingly, the prepotential H transforms as

$$e^{2H} \rightarrow e^{i\bar{\Lambda}} e^{2H} e^{-i\Lambda} \quad (3.6)$$

and

$$E^{-1} e^{\tilde{H}} \rightarrow E^{-1} e^{\tilde{H}} e^{i\bar{\Lambda}} \quad . \quad (3.7)$$

Equivalently

$$E^{-1}e^{-\overleftarrow{H}} \rightarrow E^{-1}e^{-\overleftarrow{H}}e^{i\overleftarrow{\Lambda}} \quad . \quad (3.8)$$

Indeed, we have then

$$\begin{aligned} S &\rightarrow \int d^2x d^4\theta E^{-1}e^{\overleftarrow{H}}e^{i\overleftarrow{\Lambda}} \left(e^{i\overleftarrow{\Lambda}}\bar{\phi} \right) \left(e^{i\overleftarrow{\Lambda}}e^{2H}\phi \right) \\ &= \int d^2x d^4\theta E^{-1}e^{\overleftarrow{H}}e^{i\overleftarrow{\Lambda}}e^{i\overleftarrow{\Lambda}} \left(\bar{\phi}e^{2H}\phi \right) \\ &= S \end{aligned} \quad (3.9)$$

after an integration by parts of $e^{i\overleftarrow{\Lambda}}$.

In a similar manner one establishes, by examining the transformation properties of a chiral term in the action,

$$S_{ch} = \int d^2x d^2\theta e^{-2\sigma} \left(1 \cdot e^{\overleftarrow{H}} \right) \mathcal{L}_{\text{chiral}} \quad (3.10)$$

the transformation law for the covariantly chiral compensator σ

$$e^{-2\sigma} \left(1 \cdot e^{\overleftarrow{H}} \right) \rightarrow e^{-2\sigma} \left(1 \cdot e^{\overleftarrow{H}} \right) e^{i\overleftarrow{\Lambda}} \quad . \quad (3.11)$$

It can be shown that indeed the constraints on the covariant derivatives are invariant under this set of transformations.

The Λ 's maintain the reality properties of H and are restricted by two requirements: they must be (anti)chirality-preserving, i.e. $D_{\pm}e^{i\overleftarrow{\Lambda}}\bar{\phi} = 0$ and they must maintain the vector nature of the operator $H = H^m i\partial_m$. The first requirement translates into

$$e^{-i\overleftarrow{\Lambda}}D_{\pm}e^{i\overleftarrow{\Lambda}} = a_{\pm\mp}D_{\pm} + b_{\pm\pm}D_{\mp} \quad (3.12)$$

or

$$[\bar{\Lambda}^M D_M, D_{\pm}] = 0 \quad (3.13)$$

summed over $M = (\mp, =, \dot{+}, \dot{-})$, and corresponding conditions on Λ . It leads to the relations

$$\begin{aligned} D_{\pm}\bar{\Lambda}^{\dot{+}} &= D_{\pm}\bar{\Lambda}^{\dot{-}} = D_{+}\bar{\Lambda}^{\bar{=}} = D_{-}\bar{\Lambda}^{\bar{\neq}} = 0 \\ D_{+}\bar{\Lambda}^{\bar{\neq}} &= i\bar{\Lambda}^{\dot{+}} \quad , \quad D_{-}\bar{\Lambda}^{\bar{=}} = i\bar{\Lambda}^{\dot{-}} \end{aligned} \quad (3.14)$$

which are solved by

$$\begin{aligned} \bar{\Lambda}^{\bar{\neq}} &= -D_{-}L^{\dot{+}} \quad , \quad \bar{\Lambda}^{\dot{+}} = iD^2L^{\dot{+}} \\ \bar{\Lambda}^{\bar{=}} &= -D_{+}L^{\dot{-}} \quad , \quad \bar{\Lambda}^{\dot{-}} = -iD^2L^{\dot{-}} \end{aligned} \quad (3.15)$$

in terms of an arbitrary spinor superfield $L^{\dot{\alpha}}$. Similarly, we have

$$\begin{aligned}\Lambda^{\sharp} &= -D_{\dot{-}}L^{+} \quad , \quad \Lambda^{+} = i\bar{D}^2L^{+} \\ \Lambda^{\bar{\sharp}} &= -D_{\dot{+}}L^{-} \quad , \quad \Lambda^{-} = -i\bar{D}^2L^{-} \quad .\end{aligned}\tag{3.16}$$

The second requirement translates into the following: for infinitesimal $\Lambda, \bar{\Lambda}$ we have

$$\begin{aligned}\delta e^{2H} &= i\bar{\Lambda}e^{2H} - e^{2H}i\Lambda \\ &= -\bar{\Lambda}^m\partial_me^{2H} + e^{2H}\Lambda^m\partial_m - \bar{\Lambda}^{\mu}D_{\mu}e^{2H} + e^{2H}\Lambda^{\mu}D_{\mu} - \bar{\Lambda}^{\dot{\mu}}D_{\dot{\mu}}e^{2H} + e^{2H}\Lambda^{\dot{\mu}}D_{\dot{\mu}}\end{aligned}\tag{3.17}$$

To cancel the spinor derivative terms on the right hand side we require $\Lambda^{\mu} = e^{-2H}\bar{\Lambda}^{\mu}e^{2H}$, $\Lambda^{\dot{\mu}} = e^{-2H}\bar{\Lambda}^{\dot{\mu}}e^{2H}$, etc., i.e.

$$\begin{aligned}\Lambda^{\dot{+}} &= ie^{-2H}D^2L^{\dot{+}}e^{2H} \quad , \quad \Lambda^{\dot{-}} = -ie^{-2H}D^2L^{\dot{-}}e^{2H} \\ \bar{\Lambda}^{+} &= ie^{2H}\bar{D}^2L^{+}e^{-2H} \quad , \quad \bar{\Lambda}^{-} = -ie^{2H}\bar{D}^2L^{-}e^{-2H} \quad .\end{aligned}\tag{3.18}$$

At the linearized level (where the covariantly chiral compensator is equivalent to an ordinary chiral superfield) we have for the prepotentials

$$\begin{aligned}\delta H^{\sharp} &= \frac{i}{2}(D_{\dot{-}}L^{+} - D_{-}L^{\dot{+}}) \quad , \quad \delta H^{\bar{\sharp}} = \frac{i}{2}(D_{\dot{+}}L^{-} - D_{+}L^{\dot{-}}) \\ \delta\sigma &= -\frac{i}{2}\bar{D}^2(D_{+}L^{+} - D_{-}L^{-}) \quad , \quad \delta\bar{\sigma} = -\frac{i}{2}D^2(D_{\dot{+}}L^{\dot{+}} - D_{\dot{-}}L^{\dot{-}}) \quad .\end{aligned}\tag{3.19}$$

4 Reaching light-cone gauge

Going to a specific gauge, where certain components of gauge fields are set to zero, involves examining their gauge transformations and showing that for any such transformation, $\mathcal{V} \rightarrow \mathcal{V} + \delta\mathcal{V} = \mathcal{V} + \mathcal{D}\mathcal{L}$, one can solve for the gauge parameter \mathcal{L} for any $\delta\mathcal{V}$. We go to light-cone gauge by choosing $x^{\bar{-}}$ as “time”, so that $1/\partial_{\dot{+}}$ is local and can be used when solving for gauge parameters without introducing propagating ghosts. We will show that it is possible to gauge away all of $H^{\bar{\sharp}}$ by using the gauge parameters L^{-} and $L^{\dot{-}}$, and the compensators $\sigma, \bar{\sigma}$, and certain components of H^{\sharp} by using L^{+} and $L^{\dot{+}}$. It is, of course, sufficient to examine the linearized transformations.

We consider first the transformation

$$\delta H^{\bar{\sharp}} = \frac{i}{2}(D_{\dot{+}}L^{-} - D_{+}L^{\dot{-}}) \quad .\tag{4.1}$$

We use a standard procedure to show that component by component, components of $H^{\bar{\sharp}}$ can be gauged away by components of L^{-} or $L^{\dot{-}}$ (as usual, the vertical bar

indicates evaluation at $\theta^\alpha = \theta^{\dot{\alpha}} = 0$):

$$\begin{aligned}
\delta H^\pm| &= \frac{i}{2}(D_\pm L^\mp - D_\mp L^\pm)| \\
\delta D_+ H^\pm| &= \frac{i}{2}D_+ D_\pm L^\mp| \\
\delta D_- H^\pm| &= \frac{i}{2}(D_- D_\pm L^\mp - D_\mp D_+ L^\pm)| \\
&\cdot \\
&\cdot \\
&\cdot \\
\delta[D_+, D_\pm]H^\pm| &= \frac{1}{2}\partial_\mp(D_\pm L^\mp + D_\mp L^\pm)| \\
&\cdot \\
&\cdot
\end{aligned} \tag{4.2}$$

Thus, for example (since one can invert ∂_\mp), the θ^\pm component of L^\mp and the θ^\pm component of L^\pm can be used to gauge away the first component and the $\theta^+\theta^\pm$ components of H^\pm , and so on.

It is not difficult to ascertain in this manner that H^\pm can be gauged away completely by using components of L^\mp and L^\pm . It is also easy to check that once this has been achieved one cannot use $\delta\sigma \sim \bar{D}^2 D_- L^-$ to gauge away any component of the compensators.

We turn now to the gauge transformations induced by L^+ and L^\dagger . Evidently one could use them to gauge away all of H^\pm (at the cost of introducing propagating ghosts from the inversion of the operator ∂_\pm), and this would take us to superconformal gauge. Instead, we will use them to gauge away the compensators, and some components of H^\pm .

From $\delta\sigma = -\frac{i}{2}\bar{D}^2 D_+ L^+$ it is easy to see that the four independent components of σ , namely $\sigma|$, $\bar{D}_+ \sigma|$, $D_- \sigma|$ and $D^2 \sigma|$ can be gauged away. Once this has been done, we consider the transformations $\delta H^\pm = \frac{i}{2}(D_\pm L^\mp - D_\mp L^\pm)$, but with the gauge parameters restricted now by the requirement that $\delta\sigma = \delta\bar{\sigma} = 0$, i.e.

$$\bar{D}^2 D_+ L^+ = 0 \quad , \quad D^2 D_\dagger L^\dagger = 0 \quad . \tag{4.3}$$

The first equation implies

$$D_+ \bar{D}^2 D_+ L^+ = -i\partial_\mp D_+ D_\pm L^+ = 0 \tag{4.4}$$

and since we can remove the ∂_\mp operator, $D_+(D_\pm L^+) = 0$. Together with the obvious $D_\pm(D_\pm L^+) = 0$ statement, it follows that $\mathcal{X}^\pm \equiv D_\pm L^+$ is a twisted chiral superfield.

Similarly, the second equation above implies that $\tilde{\mathcal{X}}^\sharp = D_- L^\dagger$ is a twisted antichiral superfield ². Thus, after gauging away the compensator, the residual gauge transformations of H^\sharp are

$$\delta H^\sharp = \mathcal{X}^\sharp + \tilde{\mathcal{X}}^\sharp \quad . \quad (4.5)$$

We proceed again by gauging away components of H^\sharp :

$$\begin{aligned} \delta H^\sharp| &= (\mathcal{X}^\sharp + \tilde{\mathcal{X}}^\sharp)| \\ \delta D_+ H^\sharp| &= D_+ \tilde{\mathcal{X}}^\sharp| \\ \delta D_- H^\sharp| &= D_- \mathcal{X}^\sharp| \\ \delta D_+ H^\sharp| &= D_+ \mathcal{X}^\sharp| \\ \delta D_- H^\sharp| &= D_- \tilde{\mathcal{X}}^\sharp| \\ \delta [D_+, D_+] H^\sharp| &= i \partial_+ (\mathcal{X}^\sharp - \tilde{\mathcal{X}}^\sharp)| \\ \delta [D_-, D_-] H^\sharp| &= -i \partial_- (\mathcal{X}^\sharp - \tilde{\mathcal{X}}^\sharp)| \\ \delta [D_+, D_-] H^\sharp| &= [D_+, D_-] \tilde{\mathcal{X}}^\sharp| \\ \delta [D_-, D_+] H^\sharp| &= [D_-, D_+] \mathcal{X}^\sharp| \\ \delta D^2 H^\sharp| &= 0 \\ \delta \bar{D}^2 H^\sharp| &= 0 \\ \delta \bar{D}^2 D_+ H^\sharp| &= -i \partial_+ D_- \tilde{\mathcal{X}}^\sharp| \\ \delta \bar{D}^2 D_- H^\sharp| &= i \partial_- D_+ \mathcal{X}^\sharp| \\ \delta D^2 D_+ H^\sharp| &= -i \partial_+ D_- \mathcal{X}^\sharp| \\ \delta D^2 D_- H^\sharp| &= i \partial_- D_+ \tilde{\mathcal{X}}^\sharp| \\ \delta \{D^2, \bar{D}^2\} H^\sharp| &= 0 \quad . \end{aligned} \quad (4.6)$$

It is clear then that we can gauge away all the components of H^\sharp which do not multiply θ^- , θ^\dagger , or $\theta^- \theta^\dagger$. Of the remainder, $D_- \mathcal{X}^\sharp$ appears in the transformation of two of the components of H^\sharp , namely $D_- H^\sharp|$ and $D^2 D_+ H^\sharp|$. In fact it can be used to go to a gauge where $D^2 D_+ H^\sharp| - \frac{i}{2} \partial_+ D_- H^\sharp| = 0$. A similar statement holds with respect to $D_- \tilde{\mathcal{X}}^\sharp$. It is not difficult to ascertain then that we can choose a gauge where the prepotential has the following form:

$$\begin{aligned} H^\sharp &= \theta^- \theta^\dagger [h_-^\sharp + \theta^+ \psi_-^\sharp - \theta^\dagger \psi_-^\sharp - \theta^+ \theta^\dagger D^\sharp] \\ &\quad + \theta^- [e^{\frac{i}{2} \theta^+ \theta^\dagger \partial_+} (\lambda_-^\sharp + \theta^+ N^\sharp)] + \theta^\dagger [e^{-\frac{i}{2} \theta^+ \theta^\dagger \partial_+} (\lambda_-^\sharp + \theta^\dagger \bar{N}^\sharp)] \quad . \end{aligned} \quad (4.7)$$

Thus, in light-cone gauge, the superfield H^\sharp has a natural decomposition in terms of $(2, 0)$ superfields, one of them real, the other two chiral and antichiral, with respect to D_+ and D_- . Absorbing the θ^- , θ^\dagger into these superfields, we shall write

$$H^\sharp \equiv \mathcal{H}^\sharp + \chi^\sharp + \bar{\chi}^\sharp \quad (4.8)$$

²We thank Martin Roček and Warren Siegel for suggesting this approach.

$$\mathcal{H}^\pm = \theta^- \theta^\pm \mathcal{H}_\pm^\pm \quad , \quad \chi^\pm = \theta^- \chi_\pm^\pm \quad , \quad \bar{\chi}^\pm = \theta^\pm \bar{\chi}_\pm^\pm \quad . \quad (4.9)$$

We note the following:

$$\begin{aligned} (\mathcal{H})^2 &= (\chi^\pm)^2 = (\bar{\chi}^\pm)^2 = 0 \\ \mathcal{H}^\pm \chi^\pm &= \mathcal{H}^\pm \bar{\chi}^\pm = 0 \\ \mathcal{H}^\pm D_- \chi^\pm &= -(D_- \mathcal{H}^\pm) \chi^\pm \\ \mathcal{H}^\pm D_+ \bar{\chi}^\pm &= -(D_+ \mathcal{H}^\pm) \bar{\chi}^\pm \\ D_\pm \chi^\pm &= D_\pm \bar{\chi}^\pm = 0 \quad . \end{aligned} \quad (4.10)$$

Finally, we recall that we use a notation where the corresponding differential operators would be denoted as

$$\mathcal{H} = i\mathcal{H}^\pm \partial_\mp \quad , \quad \chi = i\chi^\pm \partial_\mp \quad , \quad \bar{\chi} = i\bar{\chi}^\pm \partial_\mp \quad . \quad (4.11)$$

Corresponding to the vanishing of the superfield products above, products of such operators vanish as well.

5 (2,2) supergravity in the light-cone gauge

In this section we re-examine the solution to the (2,2) supergravity constraints discussed in ref. [7] and work out the explicit form various quantities take in light-cone gauge. We find it useful to split up e^H as follows:

$$\begin{aligned} e^H &= e^{(\mathcal{H} + \chi + \bar{\chi})} \\ &= e^{\mathcal{H}} e^{\bar{\chi}} e^{\chi} e^{-\frac{1}{2}[\bar{\chi}, \chi]} \end{aligned} \quad (5.1)$$

where we have used the Baker-Campbell-Hausdorff formula $e^{A+B} = e^A e^B e^{-\frac{1}{2}[A,B]}$ (all higher order terms being zero in this case). The commutator can also be written as

$$\begin{aligned} -\frac{1}{2}[\bar{\chi}, \chi] &= -\frac{1}{2}[i\bar{\chi}^\pm \partial_\mp, i\chi^\pm \partial_\mp] \\ &= \frac{i}{2}(\chi^\pm \overset{\leftrightarrow}{\partial}_\mp \bar{\chi}^\pm) i\partial_\mp \end{aligned} \quad (5.2)$$

so that

$$\begin{aligned} e^H &= e^{(\mathcal{H}^\pm + \frac{i}{2}\chi^\pm \overset{\leftrightarrow}{\partial}_\mp \bar{\chi}^\pm) i\partial_\mp} e^{i\bar{\chi}^\pm \partial_\mp} e^{i\chi^\pm \partial_\mp} \\ &= e^{\tilde{\mathcal{H}}^\pm i\partial_\mp} e^{i\bar{\chi}^\pm \partial_\mp} e^{i\chi^\pm \partial_\mp} \\ &= e^{\tilde{\mathcal{H}}} e^{\bar{\chi}} e^{\chi} \quad . \end{aligned} \quad (5.3)$$

We have defined

$$\tilde{\mathcal{H}}^\pm = \mathcal{H}^\pm + \frac{i}{2}\chi^\pm \overset{\leftrightarrow}{\partial}_\mp \bar{\chi}^\pm \quad (5.4)$$

and the corresponding operator.

Note that $[\tilde{\mathcal{H}}, \chi] = [\tilde{\mathcal{H}}, \bar{\chi}] = 0$ because products of these operators vanish. Therefore we can rewrite e^H as $e^{\bar{\chi}} e^{\tilde{\mathcal{H}}} e^{\chi}$ instead, and using this and the vanishing of powers of the superfields, we can derive simple expressions for the \hat{E}_α 's. We obtain:

$$\begin{aligned}\hat{E}_+ &= D_+ + iD_+(\tilde{\mathcal{H}} + \chi)^\sharp \partial_\sharp \\ \hat{E}_\dagger &= D_\dagger - iD_\dagger(\tilde{\mathcal{H}} + \bar{\chi})^\sharp \partial_\sharp \quad .\end{aligned}\tag{5.5}$$

The remaining \hat{E} 's are somewhat more complicated. We obtain

$$\begin{aligned}\hat{E}_- &= D_- + iD_-(\tilde{\mathcal{H}} + \chi)^\sharp \partial_\sharp + \chi^\sharp (\partial_\sharp D_- \chi^\sharp) \partial_\sharp + \tilde{\mathcal{H}}^\sharp (\partial_\sharp D_- \chi^\sharp) \partial_\sharp - (D_- \chi^\sharp) (\partial_\sharp \tilde{\mathcal{H}}^\sharp) \partial_\sharp \\ \hat{E}_\cdot &= D_\cdot - iD_-(\tilde{\mathcal{H}} + \bar{\chi})^\sharp \partial_\sharp + \bar{\chi}^\sharp (\partial_\sharp D_\cdot \bar{\chi}^\sharp) \partial_\sharp + \tilde{\mathcal{H}}^\sharp (\partial_\sharp D_\cdot \bar{\chi}^\sharp) \partial_\sharp - (D_\cdot \bar{\chi}^\sharp) (\partial_\sharp \tilde{\mathcal{H}}^\sharp) \partial_\sharp\end{aligned}\tag{5.6}$$

We note that since ∂_\cdot does not appear in these operators, quantities such as $\hat{C}_{+\cdot}^\pm$ and $\hat{C}_{-\dagger}^\pm$ of ref. [7] vanish, and this implies

$$A_+^- = A_{\dagger}^\cdot = 0 \quad .\tag{5.7}$$

Computing $\{\hat{E}_-, \hat{E}_\dagger\}$ we obtain $\hat{C}_{-\dagger}^\sharp$, and we find

$$\begin{aligned}A_-^+ &= i\hat{C}_{-\dagger}^\sharp \\ &= E^{-1} \left(2D_- D_\dagger \tilde{\mathcal{H}}^\sharp - 2i(D_- D_\dagger \partial_\sharp \tilde{\mathcal{H}}^\sharp) \chi^\sharp + 2i(D_- D_\dagger \tilde{\mathcal{H}}^\sharp) \partial_\sharp \chi^\sharp \right. \\ &\quad \left. + iD_- \chi^\sharp \overset{\leftrightarrow}{\partial}_\sharp D_\dagger \bar{\chi}^\sharp + \chi^\sharp (\partial_\sharp D_- \chi^\sharp) \overset{\leftrightarrow}{\partial}_\sharp D_\dagger \bar{\chi}^\sharp \right) \quad .\end{aligned}\tag{5.8}$$

From the expression in eq. (5.13) of ref. [7] it follows that in light-cone gauge $\text{sdet} \check{E}_A^B = 1$ and from eqs. (5.7), (5.10), of that reference we work out the vielbein determinant E , and its inverse, which is given by

$$E^{-1} = 1 - [D_\dagger, D_+] \tilde{\mathcal{H}}^\sharp - i\partial_\sharp (\chi - \bar{\chi})^\sharp + 2\partial_\sharp \chi^\sharp \partial_\sharp \bar{\chi}^\sharp + iD_+ \chi^\sharp \overset{\leftrightarrow}{\partial}_\sharp D_\dagger \bar{\chi}^\sharp \quad .\tag{5.9}$$

We also work out the explicit expressions

$$\begin{aligned}1 \cdot e^{-\tilde{H}} &= 1 - i\partial_\sharp \tilde{\mathcal{H}}^\sharp - i\partial_\sharp \chi^\sharp - i\partial_\sharp \bar{\chi}^\sharp - \partial_\sharp \chi^\sharp \partial_\sharp \bar{\chi}^\sharp - \chi^\sharp \partial_\sharp^2 \bar{\chi}^\sharp \\ (1 \cdot e^{-\tilde{H}})^{-1} &= 1 + i\partial_\sharp \tilde{\mathcal{H}}^\sharp + i\partial_\sharp \chi^\sharp + i\partial_\sharp \bar{\chi}^\sharp - \partial_\sharp \chi^\sharp \partial_\sharp \bar{\chi}^\sharp + \chi^\sharp \partial_\sharp^2 \bar{\chi}^\sharp \quad .\end{aligned}\tag{5.10}$$

From (2.7) it follows then that

$$e^{2\tilde{S}} = 1 + 2D_+ D_\dagger \tilde{\mathcal{H}}^\sharp + 2i\partial_\sharp \bar{\chi}^\sharp + \chi^\sharp \partial_\sharp^2 \bar{\chi}^\sharp + \partial_\sharp \chi^\sharp \partial_\sharp \bar{\chi}^\sharp + iD_+ \chi^\sharp \overset{\leftrightarrow}{\partial}_\sharp D_\dagger \bar{\chi}^\sharp \tag{5.11}$$

and

$$e^{2S} = 1 - 2D_+ D_+ \tilde{\mathcal{H}}^\dagger - 2i\partial_+ \chi^\dagger + \bar{\chi}^\dagger \partial_+^2 \chi^\dagger + \partial_+ \chi^\dagger \partial_+ \bar{\chi}^\dagger + iD_+ \chi^\dagger \partial_+ \bar{\chi}^\dagger . \quad (5.12)$$

Therefore, from the general expressions for the connections given in ref. [7] $\Omega_+ = \Omega_+ = 0$, while from

$$\Omega_- = -e^{\bar{S}} \hat{E}_+ A_-^\dagger \quad , \quad \Gamma_+ = 2e^{\bar{S}} \hat{E}_+ S \quad , \quad \Gamma_- = -e^{\bar{S}} \hat{E}_- S \quad (5.13)$$

we obtain, after some algebra,

$$\begin{aligned} \bar{R} &= -E_+ \Gamma_- - E_- \Gamma_+ - \frac{1}{2} \Omega_- \Gamma_+ \\ &= -4e^{2\bar{S}} [(\hat{E}_- \bar{S})(\hat{E}_+ S) - \hat{E}_+ \hat{E}_- S] \\ &= -4\hat{E}_- \hat{E}_+ [e^{2\bar{S}} S] . \end{aligned} \quad (5.14)$$

This expression, and a corresponding one for the conjugate, can be rewritten in the pleasing form

$$\bar{R} = 4\nabla^2 S \quad , \quad R = 4\bar{\nabla}^2 \bar{S} . \quad (5.15)$$

In terms of our explicit light-cone prepotential we find then that

$$\begin{aligned} \bar{R} &= -2e^{-i(\chi+\tilde{\mathcal{H}})\partial} D^2 [2i\partial_+ (\tilde{\mathcal{H}} + \chi)^\dagger - 4\partial_+ \chi^\dagger \partial_+ \bar{\chi}^\dagger - \chi^\dagger \partial_+^2 \bar{\chi}^\dagger - \bar{\chi}^\dagger \partial_+^2 \chi^\dagger] \\ R &= -2e^{i(\bar{\chi}+\tilde{\mathcal{H}})\partial} \bar{D}^2 [-2i\partial_+ (\tilde{\mathcal{H}} + \bar{\chi})^\dagger - 4\partial_+ \bar{\chi}^\dagger \partial_+ \chi^\dagger - \bar{\chi}^\dagger \partial_+^2 \chi^\dagger - \chi^\dagger \partial_+^2 \bar{\chi}^\dagger] . \end{aligned} \quad (5.16)$$

More compactly, in terms of the original superfield $H^\dagger = \mathcal{H}^\dagger + \chi^\dagger + \bar{\chi}^\dagger$ this can be rewritten as

$$R = 4ie^H \bar{D}^2 \partial_+ e^{-H} H^\dagger \quad , \quad \bar{R} = -4ie^{-H} D^2 \partial_+ e^H H^\dagger . \quad (5.17)$$

6 Light-cone gauge transformations

In the next section we shall derive Ward identities for correlation functions defined by (functional) averaging with the (nonlocal) induced supergravity action. They are obtained by making use of the invariance of the functional integral under a change of variables which is a field transformation. The only requirement is that under this transformation the variation of the induced action should be local. This will be the case if the field transformation is a gauge transformation for which the induced action is anomalous. In the present context this will be true for the general gauge transformations of the prepotentials, restricted however to light-cone

gauge, and chosen to preserve the form of H^\pm in (4.7). We begin by deriving these transformations. We note that, unlike the considerations of the previous section, which only required knowledge of the linearized transformations, here we need the full nonlinear transformations. Because of the nature of the light-cone prepotentials, this is not a difficult task.

We consider the general gauge transformation

$$e^{2H} \rightarrow e^{i\bar{\Lambda}} e^{2H} e^{-i\Lambda} \quad (6.1)$$

with the general form of the parameters $\Lambda, \bar{\Lambda}$ given in (3.15)-(3.18). We choose however $L^- = L^\pm = 0$, and we must suitably restrict the form of L^+ and L^\pm so as to preserve the form of H^\pm in (4.7). To see how this works we look first at the linearized level. We must restrict $\delta H^\pm = \frac{i}{2}(D_- L^+ - D_- L^\pm)$ so that

$$\delta H^\pm| = D_+ H^\pm| = D_\pm H^\pm| = [D_+, D_\pm] H^\pm| = 0 \quad (6.2)$$

and require as well that the gauge transformations preserve the (anti)chirality with respect to D_\pm , D_+ of the terms proportional to θ^- , and θ^\pm , i.e. of χ_-^\pm and χ_\pm^\pm , respectively. It follows that

$$\begin{aligned} D_- L^\pm &= \bar{\alpha}^\pm + \theta^\pm \bar{\eta}_\pm^\pm - \frac{i}{2} \theta^- \theta^\pm \partial_- \bar{\alpha}^\pm \\ D_- L^+ &= \alpha^\pm + \theta^- \eta_\pm^\pm + \frac{i}{2} \theta^- \theta^\pm \partial_- \alpha^\pm \end{aligned} \quad (6.3)$$

or

$$\begin{aligned} L^+ &= \theta^\pm (\alpha^\pm + \theta^- \eta_\pm^\pm) \\ L^\pm &= \theta^- (\bar{\alpha}^\pm + \theta^\pm \bar{\eta}_\pm^\pm) \end{aligned} \quad (6.4)$$

with $\alpha(\theta^+, \theta^\pm) = \bar{\alpha}(\theta^+, \theta^\pm)$, and $\eta_\pm^\pm(\theta^+, \theta^\pm)$ chiral. (However, this last condition will be modified at the nonlinear level). We denote $\theta^- \eta_\pm^\pm \equiv \eta_\pm^\pm$.

To obtain the full light-cone gauge transformations we start again with the Baker-Campbell-Hausdorff formula, but keeping higher order terms in the prepotentials. (However, since we only need infinitesimal transformations, we can work to first order in the gauge parameters, while terms of order higher than second in the prepotential vanish). We apply

$$e^A e^B = e^{A+B+\frac{1}{2}[A,B]+\frac{1}{12}[[A,B],B]+\frac{1}{12}[[B,A],A]} \quad (6.5)$$

twice to the expression $e^{i\bar{\Lambda}} e^{2H} e^{-i\Lambda}$. We obtain

$$e^{2H} \rightarrow e^{i(\bar{\Lambda}-\Lambda)+2H+i[\bar{\Lambda}+\Lambda,H]+\frac{i}{3}[[\bar{\Lambda}-\Lambda,H],H]} \quad (6.6)$$

Therefore,

$$\delta H = \frac{i}{2}(\bar{\Lambda} - \Lambda) + \frac{i}{2}[\bar{\Lambda} + \Lambda, H] + \frac{i}{6}[[\bar{\Lambda} - \Lambda, H], H] \quad . \quad (6.7)$$

Substituting in the expressions for $\Lambda, \bar{\Lambda}$ and H , we obtain

$$\begin{aligned} \delta H^\sharp \partial_\mp &= \frac{i}{2}(\bar{\Lambda} - \Lambda)^\sharp \partial_\mp + \frac{i}{2}(\bar{\Lambda} - \Lambda)^+ D_+ + \frac{i}{2}(\bar{\Lambda} - \Lambda)^\dagger D_\dagger - \frac{1}{2}(\bar{\Lambda} + \Lambda)^M (D_M H^\sharp) \partial_\mp \\ &+ \frac{1}{2} H^\sharp [\partial_\mp (\bar{\Lambda} + \Lambda)^\sharp] \partial_\mp + \frac{1}{2} H^\sharp [\partial_\mp (\bar{\Lambda} + \Lambda)^+] D_+ + \frac{1}{2} H^\sharp [\partial_\mp (\bar{\Lambda} + \Lambda)^\dagger] D_\dagger \\ &- \frac{i}{6} \{ (\bar{\Lambda} - \Lambda)^M (D_M H^\sharp) (\partial_\mp H^\sharp) \partial_\mp - H^\sharp \partial_\mp [(\bar{\Lambda} - \Lambda)^M D_M H^\sharp] \partial_\mp \\ &- H^\sharp [\partial_\mp (\bar{\Lambda} - \Lambda)^\sharp] (\partial_\mp H^\sharp) \partial_\mp + H^\sharp \partial_\mp [H^\sharp \partial_\mp (\bar{\Lambda} - \Lambda)^\sharp] \partial_\mp \\ &- H^\sharp [\partial_\mp (\bar{\Lambda} - \Lambda)^+] (D_+ H^\sharp) \partial_\mp + H^\sharp \partial_\mp [H^\sharp \partial_\mp (\bar{\Lambda} - \Lambda)^+] D_+ \\ &- H^\sharp [\partial_\mp (\bar{\Lambda} - \Lambda)^\dagger] (D_\dagger H^\sharp) \partial_\mp + H^\sharp \partial_\mp [H^\sharp \partial_\mp (\bar{\Lambda} - \Lambda)^\dagger] D_\dagger \} \quad . \quad (6.8) \end{aligned}$$

Since the left hand side only contains vector derivatives we require the spinorial derivative terms on the right hand side to vanish. Collecting the coefficients of D_+ and D_\dagger , respectively, we obtain the following two equations:

$$(\bar{\Lambda} - \Lambda)^+ - i H^\sharp \partial_\mp (\bar{\Lambda} + \Lambda)^+ - \frac{1}{3} H^\sharp \partial_\mp [H^\sharp \partial_\mp (\bar{\Lambda} - \Lambda)^+] = 0 \quad (6.9)$$

$$(\bar{\Lambda} - \Lambda)^\dagger - i H^\sharp \partial_\mp (\bar{\Lambda} + \Lambda)^\dagger - \frac{1}{3} H^\sharp \partial_\mp [H^\sharp \partial_\mp (\bar{\Lambda} - \Lambda)^\dagger] = 0 \quad . \quad (6.10)$$

These equations express, in light-cone gauge, the conditions given in (3.18).

Similarly, looking at the vectorial pieces (the coefficients of ∂_\mp), we find:

$$\begin{aligned} \delta H^\sharp &= \frac{i}{2}(\bar{\Lambda} - \Lambda)^\sharp - \frac{1}{2}(\bar{\Lambda} + \Lambda)^\sharp (\partial_\mp H^\sharp) - \frac{1}{2}(\bar{\Lambda} + \Lambda)^+ (D_+ H^\sharp) \\ &- \frac{1}{2}(\bar{\Lambda} + \Lambda)^\dagger (D_\dagger H^\sharp) + \frac{1}{2} H^\sharp \partial_\mp (\bar{\Lambda} + \Lambda)^\sharp \\ &- \frac{i}{6}(\bar{\Lambda} - \Lambda)^+ D_+ H^\sharp \partial_\mp H^\sharp - \frac{i}{6}(\bar{\Lambda} - \Lambda)^\dagger D_\dagger H^\sharp \partial_\mp H^\sharp \\ &+ \frac{i}{3} H^\sharp \partial_\mp (\bar{\Lambda} - \Lambda)^+ D_+ H^\sharp + \frac{i}{3} H^\sharp \partial_\mp (\bar{\Lambda} - \Lambda)^\dagger D_\dagger H^\sharp \\ &+ \frac{i}{6} H^\sharp (\bar{\Lambda} - \Lambda)^+ \partial_\mp D_+ H^\sharp + \frac{i}{6} H^\sharp (\bar{\Lambda} - \Lambda)^\dagger \partial_\mp D_\dagger H^\sharp \\ &- \frac{i}{6}(\bar{\Lambda} - \Lambda)^\sharp (\partial_\mp H^\sharp) (\partial_\mp H^\sharp) + \frac{i}{6} H^\sharp \partial_\mp (\bar{\Lambda} - \Lambda)^\sharp (\partial_\mp H^\sharp) \\ &+ \frac{i}{6} H^\sharp (\bar{\Lambda} - \Lambda)^\sharp \partial_\mp^2 H^\sharp - \frac{i}{6} H^\sharp H^\sharp \partial_\mp^2 (\bar{\Lambda} - \Lambda)^\sharp \quad . \quad (6.11) \end{aligned}$$

However, many of the terms in these expressions may be dropped because of (4.10).

We need expressions for $(\bar{\Lambda} \pm \Lambda)^M$. Using the restricted forms of L^+ , L^\dagger in (6.3), we have

$$\begin{aligned}
(\bar{\Lambda} - \Lambda)^\sharp &= [\alpha - \bar{\alpha} + \theta^- \eta_- - \theta^\dagger \bar{\eta}_+ + \frac{i}{2} \theta^- \theta^\dagger \partial_- (\alpha + \bar{\alpha})]^\sharp \\
(\bar{\Lambda} + \Lambda)^\sharp &= -[\alpha + \bar{\alpha} + \theta^- \eta_- + \theta^\dagger \bar{\eta}_+ + \frac{i}{2} \theta^- \theta^\dagger \partial_- (\alpha - \bar{\alpha})]^\sharp \\
(\bar{\Lambda} - \Lambda)^+ &= 2i[iH^\sharp \partial_\mp - \partial_\mp(\chi^\sharp \bar{\chi}^\sharp) \partial_\mp - 2\chi^\sharp \bar{\chi}^\sharp \partial_\mp^2] D_+(\alpha + \theta^- \eta_- + \frac{i}{2} \theta^- \theta^\dagger \partial_- \alpha)^\sharp \\
(\bar{\Lambda} + \Lambda)^+ &= 2i[1 + iH^\sharp \partial_\mp - \partial_\mp(\chi^\sharp \bar{\chi}^\sharp) \partial_\mp - 2\chi^\sharp \bar{\chi}^\sharp \partial_\mp^2] D_+(\alpha + \theta^- \eta_- + \frac{i}{2} \theta^- \theta^\dagger \partial_- \alpha)^\sharp \\
(\bar{\Lambda} - \Lambda)^\dagger &= -2i[-iH^\sharp \partial_\mp - \partial_\mp(\chi^\sharp \bar{\chi}^\sharp) \partial_\mp - 2\chi^\sharp \bar{\chi}^\sharp \partial_\mp^2] D_+(\bar{\alpha} + \theta^\dagger \bar{\eta}_+ - \frac{i}{2} \theta^- \theta^\dagger \partial_- \bar{\alpha})^\sharp \\
(\bar{\Lambda} + \Lambda)^\dagger &= 2i[1 - iH^\sharp \partial_\mp - \partial_\mp(\chi^\sharp \bar{\chi}^\sharp) \partial_\mp - 2\chi^\sharp \bar{\chi}^\sharp \partial_\mp^2] D_+(\bar{\alpha} + \theta^\dagger \bar{\eta}_+ - \frac{i}{2} \theta^- \theta^\dagger \partial_- \bar{\alpha})^\sharp
\end{aligned} \tag{6.12}$$

We find then

$$\begin{aligned}
\delta H^\sharp &= \frac{i}{2} \left(\alpha - \bar{\alpha} + \eta - \bar{\eta} + \frac{i}{2} \theta^- \theta^\dagger \partial_- (\alpha + \bar{\alpha}) \right)^\sharp \\
&\quad + \frac{1}{2} \left(\alpha + \bar{\alpha} + \eta + \bar{\eta} + \frac{i}{2} \theta^- \theta^\dagger \partial_- (\alpha - \bar{\alpha}) \right)^\sharp \partial_\mp H^\sharp \\
&\quad - i \left([1 + iH^\sharp \partial_\mp] D_+(\alpha + \eta)^\sharp \right) D_+ H^\sharp \\
&\quad - i \left([1 - iH^\sharp \partial_\mp] D_+(\bar{\alpha} + \bar{\eta})^\sharp \right) D_+ H^\sharp \\
&\quad - \frac{1}{2} H^\sharp \partial_\mp (\alpha + \bar{\alpha} + \eta + \bar{\eta})^\sharp .
\end{aligned} \tag{6.13}$$

Since the left hand side does not contain terms independent of θ^- , θ^\dagger , or $\theta^- \theta^\dagger$, we require

$$\alpha^\sharp - \bar{\alpha}^\sharp = 0 . \tag{6.14}$$

We have used this fact, as well as (4.10), in dropping some terms on the RHS of (6.13).

We consider separately the variation of the terms proportional to θ^- , to θ^\dagger , and to $\theta^- \theta^\dagger$. We have

$$\begin{aligned}
\delta \chi^\sharp &= \frac{i}{2} \eta^\sharp + \alpha^\sharp \partial_\mp \chi^\sharp - i D_+ \alpha^\sharp D_+ \chi^\sharp - \chi^\sharp \partial_\mp \alpha^\sharp \\
\delta \bar{\chi}^\sharp &= -\frac{i}{2} \bar{\eta}^\sharp + \alpha^\sharp \partial_\mp \bar{\chi}^\sharp - i D_+ \alpha^\sharp D_+ \bar{\chi}^\sharp - \bar{\chi}^\sharp \partial_\mp \alpha^\sharp .
\end{aligned} \tag{6.15}$$

Since the superfields χ^\sharp , $\bar{\chi}^\sharp$ are chiral and antichiral respectively, we obtain restrictions on the parameters from $D_+ \delta \chi^\sharp = D_+ \delta \bar{\chi}^\sharp = 0$, which turn out to be

$$\begin{aligned}
D_+ \left(\frac{1}{2} \eta^\sharp + i \chi^\sharp \partial_\mp \alpha^\sharp \right) &= 0 \\
D_+ \left(\frac{1}{2} \bar{\eta}^\sharp - i \bar{\chi}^\sharp \partial_\mp \alpha^\sharp \right) &= 0
\end{aligned} \tag{6.16}$$

and are solved by

$$\begin{aligned}\eta^\sharp &= -2i\chi^\sharp\partial_\mp\alpha^\sharp + D_+D_+(\chi^\sharp\alpha^\sharp) + 2D_+\gamma^\sharp_- \\ \bar{\eta}^\sharp &= 2i\bar{\chi}^\sharp\partial_\mp\alpha^\sharp - D_+D_+(\bar{\chi}^\sharp\alpha^\sharp) + 2D_+\bar{\gamma}^\sharp_- \end{aligned} \quad (6.17)$$

with arbitrary $\gamma, \bar{\gamma}$. Then

$$\begin{aligned}\delta\chi^\sharp &= iD_+[\gamma^\sharp_- + \frac{1}{2}\chi^\sharp D_+\alpha^\sharp - \frac{1}{2}\alpha^\sharp D_+\chi^\sharp] \\ \delta\bar{\chi}^\sharp &= -iD_+[\bar{\gamma}^\sharp_- + \frac{1}{2}\alpha^\sharp D_+\bar{\chi}^\sharp - \frac{1}{2}\bar{\chi}^\sharp D_+\alpha^\sharp] . \end{aligned} \quad (6.18)$$

Turning to the variation of the $\theta^-\theta^+$ terms, a certain amount of algebra leads to the conclusion that the quantity which transforms simply is not \mathcal{H}^\sharp but

$$\check{\mathcal{H}}^\sharp = \mathcal{H}^\sharp + i\chi^\sharp \overset{\leftrightarrow}{\partial}_\mp \bar{\chi}^\sharp . \quad (6.19)$$

We find

$$\delta\check{\mathcal{H}}^\sharp = -\frac{1}{2}\theta^-\theta^+\partial_\mp\alpha^\sharp + \alpha^\sharp\partial_\mp\check{\mathcal{H}}^\sharp - iD_+\alpha^\sharp D_+\check{\mathcal{H}}^\sharp - iD_+\alpha^\sharp D_+\check{\mathcal{H}}^\sharp - \check{\mathcal{H}}^\sharp\partial_\mp\alpha^\sharp . \quad (6.20)$$

Eqs. (6.18) and (6.20) represent the final form of the residual light-cone transformations.

7 Light-cone gauge Ward identities

The induced (2, 2) supergravity action has the form

$$S_{ind} = \frac{c}{4\pi} \int d^6z \bar{R} \frac{1}{\square_c} R \quad (7.1)$$

where \square_c is a suitable d'Alembertian defined, e.g., by obtaining the induced action from integrating out the covariantly chiral scalar (Goldstone) superfield in

$$S_G = -\frac{c}{4\pi} \int d^6z \bar{\Psi} \Psi + \int d^4z \Psi R + \int d^4\bar{z} \bar{\Psi} \bar{R} \quad (7.2)$$

(i.e. \square_c is the inverse of the operator $\bar{\nabla}^2 \nabla^2$ acting on a covariantly chiral superfield).

We consider correlation functions of the supergravity fields, and also of matter fields, in the presence of the induced action, defined by

$$\langle X(z_1, z_2, \dots, z_n) \rangle = \int \mathcal{D}(H, \phi) e^{S_{ind}(H) + S_m(\phi)} X(z_1, z_2, \dots, z_n) \quad (7.3)$$

where $X(z_1, z_2, \dots, z_n)$ stands for a product of n supergravity or matter fields. We work in light-cone gauge and in the functional integral make a change of integration variables which is a Λ -transformation. We assume that under this transformation S_m is invariant, while the induced action varies into the (local) anomaly. We obtain the Ward identity

$$\begin{aligned} 0 &= \int \mathcal{D}(H, \phi) e^{S_{ind}(H) + S_m(\phi)} \left[\delta S_{ind} + \sum_i \delta_i X(z_1, z_2, \dots, z_n) \right] \\ &= \langle \delta S_{ind} X(z_1, z_2, \dots, z_n) \rangle + \sum_i \langle \delta_i X(z_1, z_2, \dots, z_n) \rangle \end{aligned} \quad (7.4)$$

where $\delta_i X(z_1, z_2, \dots, z_n)$ is the variation of the i 'th field in $X(z_1, z_2, \dots, z_n)$.

In the light-cone gauge the functional integration is over the chiral and antichiral superfields χ^\pm and $\bar{\chi}^\pm$ and the real superfield \mathcal{H}^\pm , and the gauge transformations are given in (6.18) and (6.20). For the matter fields, e.g. for chiral scalar superfields with weight λ we assume transformations of the form $\delta\phi = i[\Lambda, \phi] + i\lambda(1.\bar{\Lambda})\phi$ which reduce in the present situation to $\delta\phi = -i\bar{D}^2(L^+ D_+ \phi) + i\lambda(\bar{D}^2 D_+ L^+) \phi$.

In general the induced action varies into

$$\begin{aligned} \delta S_{ind} &= \frac{ic}{4\pi} \int d^2x d^4\theta E^{-1} [R \nabla_+ L^+ + \bar{R} \bar{\nabla}_+ L^\dagger] \\ &= \frac{ic}{\pi} \int d^2x d^4\theta E^{-1} [\nabla_+ \nabla_+ S \nabla_- L^\dagger + \nabla_+ \nabla_+ \bar{S} \nabla_- L^+] \quad . \end{aligned} \quad (7.5)$$

We compute this variation explicitly, making use of the specific restricted forms of the various superfields. After some straightforward algebra we find

$$\begin{aligned} \nabla_+ \nabla_+ S &= -i D_+ D_+ \partial_\mp \mathcal{H}^\pm + \partial_\mp^2 \chi^\pm - D_+ \chi^\pm \partial_\mp^2 D_+ \bar{\chi}^\pm + D_+ \partial_\mp^2 \chi^\pm D_+ \bar{\chi}^\pm \\ &\quad - 2 D_+ \partial_\mp \chi^\pm D_+ \partial_\mp \bar{\chi}^\pm + i \partial_\mp (\partial_\mp \bar{\chi}^\pm \partial_\mp \chi^\pm) \end{aligned} \quad (7.6)$$

and

$$\begin{aligned} \nabla_+ \nabla_+ \bar{S} &= i D_+ D_+ \partial_\mp \mathcal{H}^\pm - \partial_\mp^2 \bar{\chi}^\pm - D_+ \bar{\chi}^\pm \partial_\mp^2 D_+ \chi^\pm + D_+ \partial_\mp^2 \bar{\chi}^\pm D_+ \chi^\pm \\ &\quad + 2 D_+ \partial_\mp \bar{\chi}^\pm D_+ \partial_\mp \chi^\pm + i \partial_\mp (\partial_\mp \chi^\pm \partial_\mp \bar{\chi}^\pm) \quad . \end{aligned} \quad (7.7)$$

Since these expressions explicitly contain one factor of θ^- and θ^\dagger respectively, we can simplify

$$E^{-1} \sim 1 - i \partial_\mp (\chi^\pm - \bar{\chi}^\pm) \quad . \quad (7.8)$$

We choose as our transformation parameters the superfields

$$\begin{aligned} L^+ &= \theta^\dagger (\alpha^\pm + \theta^- \eta_\pm^\pm) \\ L^\dagger &= \theta^- (\bar{\alpha}^\pm + \theta^\dagger \bar{\eta}_\pm^\pm) \end{aligned} \quad (7.9)$$

with $\alpha^\pm(\theta^+, \theta^+) = \bar{\alpha}^\pm(\theta^+, \theta^+)$ and $\eta_\pm^\pm(\theta^+, \theta^+)$, $\bar{\eta}_\pm^\pm(\theta^+, \theta^+)$ satisfying (6.17), consistent with the restriction on the transformation parameters that we have discussed earlier. Because of the explicit θ^- , θ^+ dependence there are a number of simplifications, e.g. $\nabla_\pm L^+ \sim D_\pm L^+$, etc.. After substitution in the variation of the induced action, and some algebra, we obtain

$$\begin{aligned} \delta S_{ind} = & \frac{ic}{\pi} \int d^2x d^4\theta \left\{ \left(i[D_+, D_+] \partial_\mp \mathcal{H}^\pm + \partial_\mp^2 (\chi^\pm - \bar{\chi}^\pm) \right) \alpha^\pm \right. \\ & \left. + 2\partial_\mp^2 \chi^\pm D_+ \bar{\gamma}_\pm^\pm - 2\partial_\mp^2 \bar{\chi}^\pm D_+ \gamma_\pm^\pm \right\} . \end{aligned} \quad (7.10)$$

We note that in this expression the term $\partial_\mp^2 (\chi^\pm - \bar{\chi}^\pm) \alpha^\pm$ can be dropped because it is missing either a θ^- or a θ^+ and therefore gives zero upon $d^4\theta$ integration.

We have now all the ingredients for writing down the Ward identities.

We consider, as an explicit example which can be verified in perturbation theory, the correlator $\langle \bar{\phi}(y) \phi(z) \rangle$ for an ordinary chiral scalar superfield ($\lambda = 0$). The matter lagrangian is

$$S_m = \int d^6z E^{-1} \left(e^{-H} \bar{\phi} \right) \left(e^H \phi \right) = \int d^6z (\bar{\phi} \phi - 2H^\pm D_\pm \bar{\phi} D_\pm \phi + \dots) . \quad (7.11)$$

The Ward identity becomes:

$$\langle \delta S_{ind} \bar{\phi} \phi \rangle + \langle \delta \bar{\phi} \phi \rangle + \langle \phi \delta \bar{\phi} \rangle = 0 \quad (7.12)$$

with δS_{ind} given by (7.10), and $\delta \phi$ and $\delta \bar{\phi}$ given by

$$\begin{aligned} \delta \phi &= -i \bar{D}^2 [\theta^\pm (\alpha^\pm - 2i \chi^\pm \partial_\mp \alpha^\pm + D_\pm D_\pm (\chi^\pm \alpha^\pm) + 2D_\pm \gamma_\pm^\pm) D_\pm \phi] \\ \delta \bar{\phi} &= -i D^2 [\theta^\mp (\alpha^\mp + 2i \bar{\chi}^\mp \partial_\mp \alpha^\mp - D_\mp D_\mp (\bar{\chi}^\mp \alpha^\mp) + 2D_\mp \bar{\gamma}_\mp^\mp) D_\mp \bar{\phi}] . \end{aligned} \quad (7.13)$$

Simple counting (order by order in c) reveals that terms linear in c trivially satisfy the Ward identity, terms independent of c lead to tree graphs, and terms proportional to $1/c^L$ give rise to loops. We consider here the tree graphs and note that terms dependent on α and γ must separately satisfy the Ward identity. We obtain the following equation for the α -dependent part :

$$\begin{aligned} \frac{ic}{\pi} \int d^2x d^4\theta & \langle i[D_+, D_+] \partial_\mp \mathcal{H}^\pm(x) \alpha^\pm(x) \bar{\phi}(y) \phi(z) \rangle \\ & - i \langle D^2 [\theta^\mp \alpha^\mp D_\mp \bar{\phi}(y)] \phi(z) \rangle - i \langle \bar{\phi}(y) \bar{D}^2 [\theta^\pm \alpha^\pm D_\pm \phi(z)] \rangle = 0 \end{aligned} \quad (7.14)$$

where $\langle \dots \rangle$ indicates expectation value with respect to $S_{ind} + S_m$.

In the first term we substitute the interaction term from S_m to first order in \mathcal{H}^\pm and compute

$$\begin{aligned} & -\frac{2ic}{\pi} \int d^2x d^4\theta_x d^2w d^4\theta_w \alpha^\pm(x) \cdot \\ & \langle \left(i[D_+, D_+] \partial_\mp \mathcal{H}^\pm(x) \right) \mathcal{H}^\pm(w) D_\pm \bar{\phi}(w) D_\pm \phi(w) \bar{\phi}(y) \phi(z) \rangle_0 \end{aligned} \quad (7.15)$$

Wick-contracting the \mathcal{H}^\pm 's and the ϕ 's. The second and third terms are evaluated to zeroth order in the interaction.

We need the corresponding propagators. For the matter fields we have the standard chiral propagators, e.g.

$$< \bar{\phi}(w)\phi(z) > = -\frac{D^2\bar{D}^2}{\partial_=\partial_\mp}\delta^{(2)}(w-z)\delta^{(4)}(\theta_w-\theta_z) \quad . \quad (7.16)$$

To obtain the \mathcal{H}^\pm propagator we return to the induced action which, to quadratic order, reduces to

$$S_{ind}^{(2)} = -\frac{2c}{\pi} \int d^2x d^4\theta \left(\bar{D}^2\mathcal{H}^\pm \frac{\partial_\mp}{\partial_=} D^2\mathcal{H}^\pm + \bar{\chi}^\pm \partial_\mp^2 \chi^\pm \right) \quad . \quad (7.17)$$

We consider the first term only, split off the θ^- , θ^\pm factors in $\mathcal{H}^\pm = \theta^- \theta^\pm \mathcal{H}_\pm^\pm(\theta^+ \theta^\pm)$, and perform the corresponding integration to obtain

$$S_{ind}^{(2)} = -\frac{2c}{\pi} \int d^2x d\theta^+ d\theta^\pm D_\pm \mathcal{H}_\pm^\pm D_+ \mathcal{H}_\pm^\pm = -\frac{c}{\pi} \int d^2x d\theta^+ d\theta^\pm \mathcal{H}_\pm^\pm [D_+, D_\pm] \mathcal{H}_\pm^\pm \quad . \quad (7.18)$$

in (2,0) superspace. Using $[D_+, D_\pm]^2 = -\partial_\mp^2$ we obtain for the propagator

$$< \mathcal{H}_\pm^\pm(x) \mathcal{H}_\pm^\pm(w) > = \frac{\pi}{2c} \frac{\partial_\mp}{\partial_\mp^3} [D_+, D_\pm] \delta^{(2)}(x-w) \delta(\theta_x^+ - \theta_w^+) \delta(\theta_x^\pm - \theta_w^\pm) \quad . \quad (7.19)$$

In the first term of the Ward identity we also separate off the corresponding θ 's, but instead of integrating them out we write

$$\theta_x^- \theta_x^\pm \theta_w^- \theta_w^\pm = \theta_x^- \theta_x^\pm \delta(\theta_x^- - \theta_w^-) \delta(\theta_x^\pm - \theta_w^\pm) \quad (7.20)$$

The last two δ -functions can be combined with those in the \mathcal{H}_\pm^\pm propagator leading effectively to

$$< \mathcal{H}^\pm(x) \mathcal{H}^\pm(w) > = \frac{\pi}{2c} \theta_x^- \theta_x^\pm \frac{\partial_\mp}{\partial_\mp^3} [D_+, D_\pm] \delta^{(2)}(x-w) \delta^{(4)}(\theta_x - \theta_w) \quad . \quad (7.21)$$

Now, ordinary D -algebra can be carried out in standard fashion, verifying the Ward identity which is depicted graphically in Fig.1.

Fig. 1. Diagrams representing the α Ward identity at tree-level.

A similar treatment of the terms proportional to $\bar{\gamma}$ can be used to verify the corresponding part of the Ward identity

$$\frac{2ic}{\pi} \int d^2x d^4\theta < \partial_{\mp}^2 \chi^{\pm}(x) D_+ \bar{\gamma}_{\mp}^{\pm}(x) \bar{\phi}(y) \phi(z) > -2i < D^2 [\theta^- D_+ \bar{\gamma}_{\mp}^{\pm} D_{\mp} \bar{\phi}(y)] \phi(z) > = 0 \quad (7.22)$$

To first order in the interaction we obtain for the first term

$$-\frac{4ic}{\pi} \int d^2x d^4\theta_x d^2w d^4\theta_w \cdot < \partial_{\mp}^2 \chi^{\pm}(x) D_+ \bar{\gamma}_{\mp}^{\pm}(x) \bar{\chi}^{\pm}(w) D_{\mp} \bar{\phi}(w) D_+ \phi(w) \bar{\phi}(y) \phi(z) >_0 \quad (7.23)$$

We separate out from the χ and $\bar{\chi}$ fields factors of θ^- and θ^+ and obtain the χ - $\bar{\chi}$ propagator in $(2, 0)$ superspace from (7.17):

$$< \chi_{\mp}^{\pm}(x) \bar{\chi}_{\mp}^{\pm}(w) > = -\frac{\pi i}{2c} \frac{D_{\mp} D_{\mp}}{\partial_{\mp}^3} \delta^{(2)}(x-w) \delta(\theta_x^+ - \theta_w^+) \delta(\theta_x^+ - \theta_w^+) \quad (7.24)$$

In the Ward identity we take out of the χ 's corresponding factors of θ as well as a θ_x^+ from $\bar{\gamma}$ and write $\theta_x^- \theta_x^+ \theta_w^+ = -\theta_x^- \theta_x^+ \delta(\theta_x^+ - \theta_w^+) D_- \delta(\theta_x^- - \theta_w^-)$. Combining these δ -functions with the ones in the propagator we obtain the diagrammatic representation in Fig. 2, and the Ward identity can be verified by straightforward D -algebra.

Fig. 2. Diagrams representing the $\bar{\gamma}$ Ward identity at tree-level.

8 Conclusions

In this work, starting with the prepotential formulation of $(2, 2)$ supergravity, we have used the gauge transformations of the theory to go to light-cone gauge. In this gauge the dependence on the prepotentials of the various geometrical quantities is (almost) linear and this allows us to exhibit them explicitly.

The main interest in the light-cone formulation rests on the possibility of investigating properties of the induced (due to the superconformal anomaly) nonlocal action S_{ind} , and in particular discovering the underlying “hidden” symmetry of the resulting theory, generalizing to the $(2, 2)$ case the $SL(2C)$ symmetry discovered by Polyakov for the induced gravity case [1].

We have obtained the transformation laws which are relevant to the light-cone gauge, and written down the general form of the corresponding anomalous Ward identities. However, the explicit solution of these identities, the study of the corresponding underlying algebra, and applications such as the study of β -function dressing [4, 6] are left to a future publication.

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